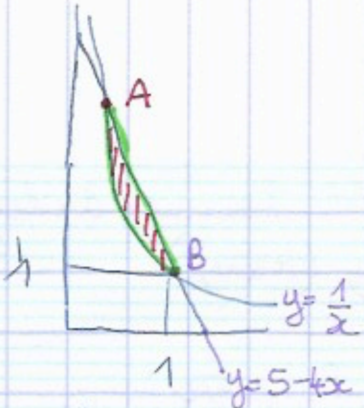
⑥ $\iint_D x^2 y \, dx\, dy$



clairement $B(1,1)$

quant à A, son abscisse vérifie $\frac{1}{x} = 5-4x$

soit $1 = 5x - 4x^2$

$\Leftrightarrow 4x^2 - 5x + 1 = 0$

$\Delta = 9$ $\quad x = \frac{5 \pm 3}{8} = 1$ ou $\frac{1}{4}$: $x_A = \frac{1}{4}$

donc $I = \int_{x=\frac{1}{4}}^{1} \int_{y=\frac{1}{x}}^{5-4x} x^2 y \, dy\, dx$

$= \frac{1}{2} \int_{x=\frac{1}{4}}^{1} x^2 \left[y^2 \right]_{y=1/x}^{5-4x} dx$

$= \frac{1}{2} \int_{x=\frac{1}{4}}^{1} x^2 \left\{ (5-4x)^2 - \left(\frac{1}{x}\right)^2 \right\} dx$

$= \frac{1}{2} \int_{x=1/4}^{1} \left(-1 + 25x^2 - 40x^3 + 16x^4 \right) dx$

$= \frac{1}{2} \left[-x + \frac{25}{3} x^3 - 10x^4 + 3{,}2x^5 \right]_{1/4}^{1}$

$= \frac{1}{2} \left(-1 + \frac{25}{3} - 10 + 3{,}2 + \frac{1}{4} - \frac{25}{3 \times 64} + 10 \frac{1}{256} - \frac{32}{10} \times \frac{1}{2^{10}} \right)$

$= \frac{1}{2} \left(-7{,}8 + \frac{25}{3} + \frac{16}{64} - \frac{25/3}{64} + \frac{2{,}5}{64} - \frac{0{,}2}{64} \right)$

$= \frac{1}{2} \left(\frac{1{,}6}{3} + \frac{10{,}3 - 1/3}{64} \right) \approx 0{,}345$

$=$